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Chiral radiative corrections and the $D_s(2317)/D(2308)$ mass puzzle

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Abstract. We show that the one-loop chiral corrections for heavy light mesons in the potential model can explain the small mass of $D_s(2317)$ as well as the small mass gap between $D_s(2317)$ and $D(2308)$.

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The recently observed $D_s(2317)$ [1–3], which is a very narrow resonance ($\Gamma < 10\,\text{MeV}$) decaying into $D_s^+\pi^0$, is thought to be the missing bound state with $J^P = 0^+$ of the heavy–light system. This picture of $D_s(2317)$ composed of a heavy quark and a light valence quark fits well with the heavy quark, chiral symmetries that predict parity doubling states $(0^-, 1^-)$ and $(0^+, 1^+)$, with the interparity mass splittings in the chiral limit given by the Goldberger–Treiman relation [4–6]. The subsequent observation of 1^+ state $D_s(2460)$ [1–3] strongly supports this picture.

On the other hand, the two-quark picture of the resonances does not play well with the potential model calculations, which generally predict substantially larger mass and width. According to the potential model calculation in [7] the mass and width of $D_s(0^+)$ are, respectively, 2487 MeV and a few 100 MeV, with the width depending on the light quark axial coupling. While the narrow decay width can be understood by the observed mass being below the threshold of the strong decay channel DK and the isospin symmetry breaking, the substantially small observed mass is puzzling.

Furthermore, this anomaly in the observed mass became more peculiar when the Belle collaboration observed [8] the non-strange 0^+ state $D(2308)$, whose mass is surprisingly close to $D_s(2317)$. The potential model predicts the mass splitting between these states to be 110 MeV. These peculiarities in the observed masses led to many models for the new resonances, including, for example, a four-quark model [9, 10], DK molecule models [11], and a unitarized meson model [12]. It is thus very important to clarify the nature of the newly discovered resonances.

The quoted numbers of the potential model calculation are based on a Coulombic vector potential and a linear scalar potential. Modifications of the employed potentials might remove the anomaly, but Cahn and Jackson [13] showed that, as far as the vector potential is kept Coulombic, it is unlikely that the observed decay width and mass pattern of the resonances can be obtained from a potential model.

This suggests that the potential model be missing essential physics of the heavy–light system. Indeed, the conventional potential model does not sufficiently take into account the chiral symmetry breaking nature of the QCD vacuum, with the chiral symmetry breaking encoded only in the light quark constituent masses of the model. Since the light valence quark is chirally active, the heavy–light mesons can couple to the quantum fluctuations of the Goldstone bosons of the QCD vacuum. This suggests that potential models must be augmented by chiral radiative corrections.

In this paper we calculate chiral radiative corrections for the bound state energies of the potential model, paying particular attention to the mass splittings of the parity doubling states. Our main result is that chiral corrections are large, comparable at least to $1/M_c$ corrections in the D mesons, where M_c denotes the charm mass, rendering their inclusion in the potential model mandatory. Furthermore, for the parity doubling states, they tend to narrow the interparity mass gaps, and this effect is stronger in a strange system than in a non-strange system, with the robust prediction of the mass gap $\equiv [m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))] \approx 90\,\text{MeV}$ (at the axial coupling $g_A = 0.82$) that is consistent with experiment.

The potential model of the heavy–light system [14] is based on the chiral quark model [15], with the Lagrangian

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reading

$$\mathcal{L} = \Psi^\dagger(\mathrm{i}\partial_0 - H)\Psi\,, \tag{1}$$

with $\Psi = (u, d, s)$ denoting the light quark fields and the Hamiltonian given by

$$H = H_0 + \frac{1}{M}H_1 + \cdots\,, \tag{2}$$

where M denotes the heavy quark mass. The leading Hamiltonian H_0 in the heavy quark mass expansion reads

$$H_0 = \gamma^0(-\mathrm{i}\,\nabla\!\!\!/ + \mathbf{m}) + V(r), \tag{3}$$

with the potential given in the form

$$V(r) = M + \gamma^0 V_s(r) + V_\mathrm{v}(r)\,, \tag{4}$$

where V_s and V_v denote the scalar and vector potentials, respectively, and $\mathbf{m} = m_i\delta_{ij}$ denotes the constituent quark masses. The energy spectra of resonances are obtained by solving the Dirac equation of H_0, followed by time-independent perturbations of the subleading terms. The free parameters of the model are fixed by a global fitting of the predicted masses to those of the observed resonances.

In this framework the chiral symmetry breaking of QCD is encoded only in the constituent masses of the light quarks, and we shall see that this is not sufficient. This inadequacy of the model can easily be remedied by noting that the effective Hamiltonian is based on a truncated chiral quark model. In chiral quark model the light quark–Goldstone boson interactions are described by an infinite tower of derivative expansions, but the term responsible for the one-loop corrections is the following axial coupling:

$$\begin{aligned} H_{\bar\psi\psi\pi} &= -g_\mathrm{A}\bar\Psi A\!\!\!/\gamma_5\Psi \\ &= \frac{g_\mathrm{A}}{2f_\pi}\bar\Psi_i\gamma^\mu\gamma_5\Psi_j\partial_\mu\Pi_{ij} + O(\Pi^2), \end{aligned} \tag{5}$$

where g_A is an axial coupling constant, and

$$A_\mu = \frac{\mathrm{i}}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right), \tag{6}$$

with $\xi = \mathrm{e}^{\mathrm{i}\Pi/2f_\pi}$, where $\Pi = \sum_{a=1}^{8}\pi^a\lambda^a$, λ^a the Gell-Mann matrices, and $f_\pi = 93\,\mathrm{MeV}$.

We note that the inclusion of the axial term (5) in the potential model Hamiltonian should not be unexpected, since this term was already employed in the calculation of the decay widths in the potential model. In general the widths, which are the imaginary parts of the self-energies, can be a few hundred MeV, which indicates that the chiral radiative corrections to the resonance masses cannot be small, and so should be included in the computation of the masses.

We shall now consider the corrections due to the chiral term (5) to the energy of an eigenstate of H_0. Let us denote the eigenenergy and normalized wavefunction by $E_\mathbf{m}$ and $\Psi_\mathbf{m}$, respectively. Here $\mathbf{m} = \{n, l, j, m_j, q\}$ denotes the set of quantum numbers classifying the eigenstate of the light quark, with n, q, and l, j, m_j denoting the radial excitation, quark flavor, and the angular momentum quantum numbers, respectively. The correction to the energy $E_\mathbf{m}$ at one loop comes through the diagram in Fig. 1 and is given by

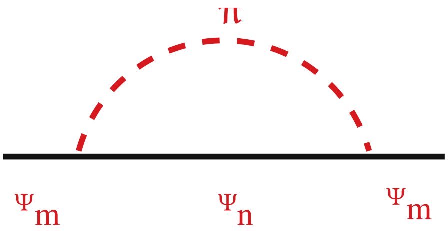

Fig. 1. One loop correction to the energy of the eigenstate $\Psi_\mathbf{m}$

$$\Delta E_\mathbf{m} = \frac{ig_\mathrm{A}^2}{4f_\pi^2}\sum_\mathbf{n}\sum_\pi\zeta_\pi \times\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{|j_\mathbf{mn}(k)|^2}{(E_\mathbf{m} - k_0 - E_\mathbf{n} + \mathrm{i}\epsilon)\,(k^2 - m_\pi^2 + \mathrm{i}\epsilon)}\,, \tag{7}$$

where

$$j_\mathbf{mn}(k) = \left(\int\mathrm{d}^3\mathbf{x}\;\Psi_\mathbf{m}^\dagger(\mathbf{x})\gamma^0\gamma^\mu\gamma_5\Psi_\mathbf{n}(\mathbf{x})\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}\right)k_\mu\,, \tag{8}$$

and m_π denotes the mass of the light meson exchanged and ζ_π represents the $SU(3)_\mathrm{flavor}$ factors coming from the axial vertices.

Using the Dirac equations for the wavefunctions the current can be written as

$$j_\mathbf{mn}(k) = (k_0 - E_\mathbf{m} + E_\mathbf{n})\rho^{(1)}_\mathbf{mn}(\mathbf{k}) + \rho^{(2)}_\mathbf{mn}(\mathbf{k})\,, \tag{9}$$

where

$$\begin{aligned} \rho^{(1)}_\mathbf{mn}(\mathbf{k}) &= \int\mathrm{d}^3\mathbf{x}\;\Psi_\mathbf{m}^\dagger(\mathbf{x})\gamma_5\Psi_\mathbf{n}(\mathbf{x})\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}\,, \\ \rho^{(2)}_\mathbf{mn}(\mathbf{k}) &= \int\mathrm{d}^3\mathbf{x}\;\Psi_\mathbf{m}^\dagger(\mathbf{x})\gamma^0\gamma_5(m_\mathbf{m} \\ &\quad + m_\mathbf{n} + 2V_s)\Psi_\mathbf{n}(\mathbf{x})\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}\,. \end{aligned} \tag{10}$$

Substituting (9) into (7), and performing the integration over k_0 we obtain

$$\Delta E_\mathbf{m} = \sum_\mathbf{n}\sum_\pi\zeta_\pi J(\mathbf{m}, \mathbf{n}, m_\pi)\,, \tag{11}$$

where

$$\begin{aligned} J(\mathbf{m}, \mathbf{n}, m_\pi) = \frac{-g_\mathrm{A}^2}{8f_\pi^2}&\int\frac{\mathrm{d}^3k}{(2\pi)^3E_\pi(\mathbf{k})} \\ &\times\Bigg[(E_\mathbf{n} - E_\mathbf{m})|\rho^{(1)}_\mathbf{mn}(\mathbf{k})|^2 \\ &+ 2\,\mathrm{Re}\left[\rho^{(1)}_\mathbf{mn}(\mathbf{k})\rho^{(2)}_\mathbf{mn}(\mathbf{k})^*\right] \\ &+ \frac{|\rho^{(2)}_\mathbf{mn}(\mathbf{k})|^2}{E_\pi(\mathbf{k}) - E_\mathbf{m} + E_\mathbf{n} - \mathrm{i}\epsilon}\Bigg], \end{aligned} \tag{12}$$

with $E_\pi(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_\pi^2}$.

We shall now write the currents $\rho^{(i)}_{\mathbf{mn}}$ in terms of the radial functions of the eigenfunctions, which can be written as

$$\Psi_{\mathbf{m}}(\mathbf{r}) = \begin{pmatrix} \mathrm{i} f_{n\ell jq}(r) \\ g_{n\ell jq}(r)\sigma\cdot\hat{r} \end{pmatrix} \mathcal{Y}_{\ell jm}(\hat{r}), \quad (13)$$

where $\mathcal{Y}_{\ell jm}(\hat{r})$ is the spinor harmonics.

Since the light quark wavefunctions are eigenstates of the angular momentum operator, it is convenient to expand the plane wave $\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})$ in the definition of the currents $\rho^{(1,2)}_{\mathbf{mn}}$ in (10) as

$$\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} = 4\pi \sum_{\ell=0}^{\infty} i^{\ell} j_{\ell}(kr) \sum_{m=-\ell}^{+\ell} Y^{*}_{\ell,m}(\hat{r})\, Y_{\ell,m}(\hat{k})\,. \quad (14)$$

Then, the currents $\rho^{(1,2)}_{\mathbf{mn}}$ can be expanded as

$$\rho^{(1,2)}_{\mathbf{mn}}(\mathbf{k}) = \sum_{\ell,m} \rho^{(1,2)}_{\mathbf{mn}}(|\mathbf{k}|,\ell,m) Y_{\ell,m}(\hat{k}), \quad (15)$$

with, up to a common phase,

$$\begin{aligned}\rho^{(1,2)}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi},m_{\pi}) &= 4\pi \int_0^{\infty} r^2 \mathrm{d}r \tilde{\rho}^{(1,2)}_{\mathbf{mn}}(r) j_{l_{\pi}}(kr) \\ &\quad \times \langle j m_j l_{\pi} m_{\pi} | j' m'_j \rangle \langle \ell j || Y^{*}_{l_{\pi}} \sigma\cdot\hat{r} || \ell' j' \rangle \\ &\equiv \langle j m_j l_{\pi} m_{\pi} | j' m'_j \rangle\, \rho^{(1,2)}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi})\,, \end{aligned} \quad (16)$$

where

$$\begin{aligned}\tilde{\rho}^{(1)}_{\mathbf{mn}} &= f_{\mathbf{m}}(r) g_{\mathbf{n}}(r) - f_{\mathbf{n}}(r) g_{\mathbf{m}}(r)\,, \\ \rho^{(2)}_{\mathbf{mn}} &= \big(f_{\mathbf{m}}(r) g_{\mathbf{n}}(r) + f_{\mathbf{n}}(r) g_{\mathbf{m}}(r)\big)(m_{\mathbf{m}} + m_{\mathbf{n}} + 2V_s)\,. \end{aligned} \quad (17)$$

Here $\{\ell, j, m_j\}$ are the angular momentum quantum numbers of the state $\mathbf{m}$, $\{\ell', j', m'_j\}$ those of the state $\mathbf{n}$ and ℓ_{π} is the angular momentum of the intermediate meson appearing in the expansion (14), and $\langle \ell j || Y^{*}_{l_{\pi}} \sigma\cdot\hat{r} || \ell' j' \rangle$ denotes the reduced matrix element. Equation (16) provides a selection rule for the possible intermediate light meson angular momentum, l_{π}, for a given internal state and vice versa.

Now, doing the angular part of the $\mathbf{k}$-integration, which can be easily carried out with the decomposition (15), and using the unitarity relation

$$\sum_{m'_j} \sum_{m_{\pi}+m_j=m'_j} \langle j m_j l_{\pi} m_{\pi} | j' m'_j \rangle^2 = \sum_{m'_j} \{1\} = 2j'+1, \quad (18)$$

we can rewrite the loop corrections to the energy as

$$\Delta E_{\mathbf{m}} = \sum_{\mathbf{n}} \sum_{\pi, l_{\pi}} \zeta_{\pi} J(\mathbf{m},\mathbf{n},l_{\pi}) \frac{2j_{\mathbf{n}}+1}{2j_{\mathbf{m}}+1}, \quad (19)$$

where

$$\begin{aligned} J(\mathbf{m},\mathbf{n},l_{\pi}) = -\frac{g_{\mathrm{A}}^2}{8f_{\pi}^2} \int \frac{k^2 \mathrm{d}k}{(2\pi)^3 E_{\pi}} \Bigg[&(E_{\mathbf{n}} - E_{\mathbf{m}}) |\rho^{(1)}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi})|^2 \\ &+ 2\,\mathrm{Re}[\rho^{(1)}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi}) \rho^{(2)*}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi})] \\ &+ \frac{|\rho^{(2)}_{\mathbf{mn}}(|\mathbf{k}|,l_{\pi})|^2}{E_{\pi} - E_{\mathbf{m}} + E_{\mathbf{n}} - \mathrm{i}\epsilon} \Bigg]. \end{aligned} \quad (20)$$

We now focus on the energy corrections for the lowest energy parity doubling states, $D(0^-), D_s(0^-)$ and $D(0^+), D_s(0^+)$. They have the quantum numbers $\mathbf{m} = \{1, 0, \frac{1}{2}, \pm\frac{1}{2}, q=(d,s)\}$, $\{1, 1, \frac{1}{2}, \pm\frac{1}{2}, q=(d,s)\}$, and, to shorten the notation, these will be denoted $\mathbf{0}_{d,s}, \mathbf{1}_{d,s}$, respectively. For these states $\rho^{(i)}_{\mathbf{m},\mathbf{n}}(|\mathbf{k}|,\ell_{\pi})$ in (20) are given by

$$\begin{aligned} \rho^{(1)}_{\mathbf{m},\mathbf{n}}(|\mathbf{k}|,\ell_{\pi}) = \sqrt{4\pi} \int_0^{\infty} r^2 \mathrm{d}r (&f_{\mathbf{m}}(r) g_{\mathbf{n}}(r) \\ &- f_{\mathbf{n}}(r) g_{\mathbf{m}}(r)) j_{\ell_{\pi}}(kr), \end{aligned} \quad (21)$$

$$\begin{aligned} \rho^{(2)}_{\mathbf{m},\mathbf{m}}(|\mathbf{k}|,\ell_{\pi}) = \sqrt{4\pi} \int_0^{\infty} r^2 \mathrm{d}r (&f_{\mathbf{m}}(r) g_{\mathbf{n}}(r) + f_{\mathbf{n}}(r) g_{\mathbf{m}}(r)) \\ &\times (m_{\mathbf{m}} + m_{\mathbf{n}} + 2V_s) j_{\ell_{\pi}}(kr)\,. \end{aligned} \quad (22)$$

Before giving the numerical result, we comment on the divergence of the loop corrections. The loop correction $J(\mathbf{m},\mathbf{n},l_{\pi})$ for given $\mathbf{m},\mathbf{n}$ is free from ultraviolet (UV) divergences, with the wavefunctions providing the UV cutoff. However, the total loop correction obtained by summing over the internal states is quadratically divergent. The quadratic divergence comes from the first two terms of (12), which can be easily summed over the internal states using the Dirac equation for the wavefunctions and the definition of $\rho^{(i)}_{\mathbf{mn}}$ in (10). This gives the sum of the first two terms as

$$\begin{aligned} \Delta E^{\text{quad. div.}}_{\mathbf{m}} = \sum_{\pi} \zeta_{\pi} \frac{-g_{\mathrm{A}}^2}{8f_{\pi}^2} &\left[\int \frac{\mathrm{d}^3 k}{(2\pi)^3 E_{\pi}(\mathbf{k})} \right] \\ &\times \int \mathrm{d}^3 x\, \Psi^{\dagger}_{\mathbf{m}} \gamma^0 (m_{\mathbf{m}} + m_{\mathbf{n}} + 2V_s) \Psi_{\mathbf{m}}\,. \end{aligned} \quad (23)$$

The third term in (12) is at most linearly divergent. This quadratic divergence of the energy correction is not unexpected since the chiral quark model is an effective theory valid only at low energies. To regularize the UV divergence we introduce a three-momentum cutoff of the form

$$\mathrm{e}^{-\mathbf{k}^2/\Lambda^2_{\mathrm{UV}}} \quad (24)$$

to the integrand in (20). We regard Λ_{UV} as the physical cutoff of the chiral quark model, but we shall see that our main result on the mass gap depends little on the cutoff.

To obtain the eigenfunctions of H_0 we should first fix the parameters of the model. In the following we shall follow the setup as well as use the parameter values given in [7], in which the vector potential is Coulombic and the scalar potential is a linearly confining potential. The model has nine free parameters that are to be fixed by a global fitting of the predicted resonance masses to those observed values. For details we refer the readers to the above reference. Of course, the parameters were fixed without taking the chiral corrections into account, but we can use those values to estimate the loop correction effects at leading order, which are comparable in magnitude to the $1/M_c$ corrections and so play only a subleading role in fitting the parameters.

We also need to fix the axial coupling g_{A}, which is a free parameter in the chiral quark model. In principle it can be determined by fitting the hadronic decay width of an excited heavy–light meson to the experimental data, or by lattice simulation. These approaches estimate g_{A} to be around unity with a considerable uncertainty [7].

It is convenient to organize the energy corrections in terms of the angular momentum l_π of the intermediate light mesons. For a given l_π we sum over all allowed internal states, which can be selected by the angular momentum and parity conservations at the axial vertex, up to the first 10 radial excitations. As can be seen in Table 1 the corrections drop rapidly at higher radial excitations. Our result is summarized in Table 2 at varying cutoffs Λ_{UV}. At a smaller cutoff $\Lambda_{\mathrm{UV}} = 700$ MeV the corrections drop quickly as l_π increases, whereas at a larger cutoff $\Lambda_{\mathrm{UV}} = 1200$ MeV they drop slowly, reflecting the UV divergence of the chiral corrections. At all the cutoffs considered, the largest contributions come from the $l_\pi = 1$ modes, and at larger cutoffs contributions from l_π as large as four are significant.

Not surprisingly, the total energy corrections depend strongly on the UV cutoff. At $\Lambda_{\mathrm{UV}} = 700$ MeV they are a few hundred MeV but at $\Lambda_{\mathrm{UV}} = 1200$ they are in GeV order. This shows that in our model the physical cutoff should be about 700 MeV. Although the total corrections are sensitive on the cutoff, we expect the difference of the interparity mass gaps between the parity doubling states is less sensitive to the cutoff. Indeed, summing the contributions up to $l_\pi = 9$ we find that

$$\Delta E\mathbf{1}_d - \Delta E\mathbf{0}_d = -146, -312, -447\,\mathrm{MeV}\,,$$
$$\Delta E\mathbf{1}_s - \Delta E\mathbf{0}_s = -271, -442, -579\,\mathrm{MeV}\,, \qquad (25)$$

and

$$\mathrm{gap} \equiv (\Delta E\mathbf{1}_d - \Delta E\mathbf{0}_d) - (\Delta E\mathbf{1}_s - \Delta E\mathbf{0}_s)$$
$$= 125, 130, 132\,\mathrm{MeV} \qquad (26)$$

at $\Lambda_{\mathrm{UV}} = 700, 1000, 1200$ MeV, respectively, and $g_{\mathrm{A}} = 1$. This shows that the chiral corrections shrink the interparity gaps, both in strange and non-strange systems, but do so more in the strange system. This may be an explanation for the mass of $D_s(2317)$ being unusually smaller than given in the potential model. More interestingly, the mass gap is remarkably stable under variation of the cutoff. This suggests that the gap in (26) comes almost entirely from the low energy region far down the cutoff. To see this we plot in Fig. 2 the differential gap $\mathcal{G}(k)$, defined as the gap before the integration over the momentum variable k, that is,

$$\mathrm{gap} \equiv \int_0^\infty \mathcal{G}(k)\mathrm{d}k\,,$$

which can be obtained from the proper combination of the integrands for the energy corrections given in (20). The discontinuity in the plot comes from our implementation of the principal value prescription for the last term in the integrand in (20) which has a pole at $k_{\mathrm{p}} = \sqrt{(E_{\mathbf{m}} - E_{\mathbf{n}})^2 - m_\pi^2}$ when the external state is $\mathbf{1}_d$ and the internal state is $\mathbf{0}_d$ ($E_{\mathbf{m}} = 2282$ MeV, $E_{\mathbf{n}} = 1895$ MeV, $m_\pi = 140$ MeV). Our numerical code handles the principal value

Table 1. Energy corrections from the first five radial excitations at $l_\pi = 0$. The gap $\equiv (\Delta E\mathbf{1}_d - \Delta E\mathbf{0}_d) - (\Delta E\mathbf{1}_s - \Delta E\mathbf{0}_s)$. Values are at $\Lambda_{\mathrm{UV}} = 700$ MeV and $g_{\mathrm{A}} = 1$. Units are in MeV

n	1	2	3	4	5
$\Delta E\mathbf{0}_d$	−23	−3	0	0	0
$\Delta E\mathbf{1}_d$	29	−49	−8	−1	0
$\Delta E\mathbf{0}_s$	−33	−3	0	0	0
$\Delta E\mathbf{1}_s$	−108	−49	−9	−1	0
gap	127	0	1	0	0

Table 2. Energy corrections ($g_{\mathrm{A}} = 1$) versus l_π. The gap $\equiv (\Delta E\mathbf{1}_d - \Delta E\mathbf{0}_d) - (\Delta E\mathbf{1}_s - \Delta E\mathbf{0}_s)$. Units are in MeV

	l_π	0	1	2	3	4	5	6	7	8	9
	$\Lambda_{\mathrm{UV}} = 700$	−26	−142	−50	−16	−5	−1	0	0	0	0
$\Delta E\mathbf{0}_d$	$\Lambda_{\mathrm{UV}} = 1000$	−116	−361	−196	−93	−41	−17	−7	−3	−1	0
	$\Lambda_{\mathrm{UV}} = 1200$	−201	−561	−362	−202	−105	−52	−25	−12	−6	−3
	$\Lambda_{\mathrm{UV}} = 700$	−29	−192	−101	−42	−15	−5	−2	0	0	0
$\Delta E\mathbf{1}_d$	$\Lambda_{\mathrm{UV}} = 1000$	−64	−400	−310	−185	−98	−49	−23	−11	−5	−2
	$\Lambda_{\mathrm{UV}} = 1200$	−93	−558	−502	−349	−215	−124	−69	−37	−19	−10
	$\Lambda_{\mathrm{UV}} = 700$	−37	−136	−47	−14	−4	0	0	0	0	0
$\Delta E\mathbf{0}_s$	$\Lambda_{\mathrm{UV}} = 1000$	−141	−376	−197	−88	−37	−15	−6	−2	0	0
	$\Lambda_{\mathrm{UV}} = 1200$	−239	−599	−372	−197	−97	−46	−21	−10	−4	−2
	$\Lambda_{\mathrm{UV}} = 700$	−168	−186	−98	−39	−14	−4	−1	0	0	0
$\Delta E\mathbf{1}_s$	$\Lambda_{\mathrm{UV}} = 1000$	−217	−413	−317	−183	−94	−45	−20	−9	−4	−2
	$\Lambda_{\mathrm{UV}} = 1200$	−253	−589	−524	−354	−210	−117	−63	−32	−16	−8
	$\Lambda_{\mathrm{UV}} = 700$	128	0	0	−1	0	0	−1	0	0	0
gap	$\Lambda_{\mathrm{UV}} = 1000$	128	−2	6	3	0	−2	−2	−1	0	0
	$\Lambda_{\mathrm{UV}} = 1200$	122	−7	12	10	3	−1	−2	−3	−1	−1

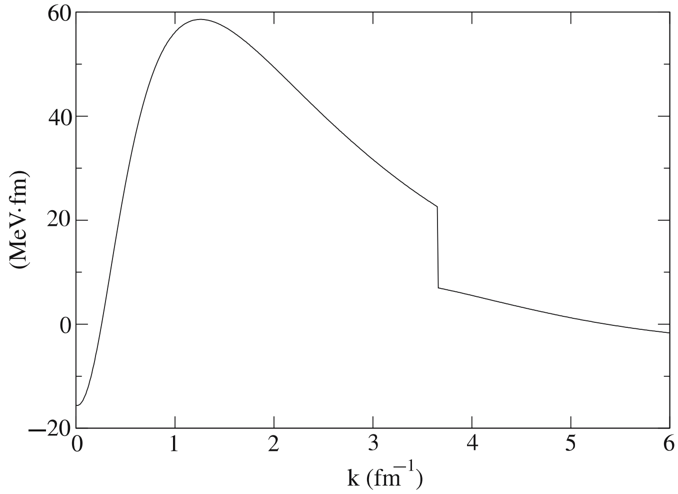

Fig. 2. The differential gap $\mathcal{G}(k)$. No cutoff applied ($\Lambda_{\rm UV} = \infty$), and $g_{\rm A} = 1$. Notice that the bulk of the contribution to the gap comes from the low energy region

integration using the identity

$$\int_0^\infty \frac{f(k)}{k-k_{\rm p}} {\rm d}k = \int_0^{2k_{\rm p}} \frac{f(k)-f(k_{\rm p})}{k-k_{\rm p}} {\rm d}k + \int_{2k_{\rm p}}^\infty \frac{f(k)}{k-k_{\rm p}} {\rm d}k, \tag{27}$$

which is valid for any smooth function $f(k)$. With this the pole at $k_{\rm p}$ is now removed from the integrand and there appears a discontinuity at $2k_{\rm p}$. Notice that the differential gap has a peak around $k \approx 250$ MeV and the bulk of the contribution to the gap comes from the low energy region. This shows that our prediction of the gap is not affected by the UV physics and thus is safe from the truncation of the higher derivative terms in the chiral quark model.

Without the chiral corrections the potential model predicts an almost vanishing gap, while experimentally it is about 95 MeV. If we take $g_{\rm A} = 0.82$, as given in [7], we get the experimentally consistent value of 90 MeV for the gap. We note that a similar result was observed by Eichten using the heavy–light chiral Lagrangian [16], but this was obtained without taking into account the tree-level mass terms for the heavy–light mesons arising from the explicit chiral symmetry breaking.

It is notable that the gap is dominated by the contributions from the $n = 1, l_\pi = 0$ modes, as can be seen from Tables 1 and 2. Interestingly, these modes widen the interparity mass gap in a non-strange system whereas in a strange system they narrow the gap, but still the gap from these modes is already consistent with experiment.

In conclusion, we calculated the one-loop chiral corrections for the heavy–light mesons in the potential model based on the truncated chiral quark model, and we have shown that the chiral corrections can account for the unusually small mass of $D_s(2317)$ and the narrow mass difference between $D_s(2317)$ and $D(2308)$. Our calculation strongly supports the two-quark picture of the new resonances of being composed of a heavy quark and a light valence quark.

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